FOURIER TRANSFORMED MATRIX METHOD OF FINDING PROPAGATION CHARACTERISTICS OF COMPLEX ANISOTROPIC LAYERED MEDIA

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Abstract

A structure having arbitrarily located conductor lines immersed in complex anisotropic layered media presents one with a very general guided wave problem. This problem is solved here by a rigorous formulation technique characterizing each layer by a 6 × 6 constitutive tensor and finding the appropriate Fourier transformed Green's function matrix G. From G a method of moments solution for the propagation characteristics follows including propagation constant eigenvalues and field eigenvectors at all spatial locations.

Introduction

Advances in materials technology are allowing the contiguous growth of substances of considerably different properties. Present integrated circuit processing techniques allow various combinations of metals, dielectrics, and semiconductors to be layered together where these materials may or may not be crystalline. We may expect to see in the future the use of magnetic films [1] (metallic or non-metallic), uniaxial and biaxial dielectric films [2], ferrite films, magnetically induced semiconductor gyroelectric films, and widely varying compositions of compound films such as binary, ternary, and quaternary compounds. More creative use of materials, especially for monolithic integrated circuits, will probably occur in the future. Very complex materials with a combination of birefringent gyroelectric, gyromagnetic, optical rotation, or other anisotropic properties may be utilized. Furthermore the use of semiconductors, dielectrics, or magneto-materials rotated off principal axes or convenient axis coordinates can be envisioned.

Conventional methods either in direct or Fourier transformed space using planar symmetry are not general enough to enable the interested worker in the microwave or millimeter wave area to readily solve such complex problems outlined above. Expeditious ways of solving field problems based on Maxwell's equations, but avoiding gauge methods, are possible by using field matrix techniques. Matrix techniques using two field components have been often used in the optics [3] and microwave/electromagnetics [4] areas. As the medium becomes more complex with less symmetry, the 2-component methods become increasingly difficult to implement. Lack of conductor line symmetry also complicates the 2-component solution methods. A 4-component method has the great advantage of enabling the use of only 1st order partial differential equations. The 4-component technique also has the ability to allow direct field matching at layer boundaries or interfaces [5].

Here a new formulation technique for solving the uniform (in the z-direction) waveguide propagation problem is developed for layered media possessing complex anisotropic properties. A 4-component method is utilized by adapting the 4 × 4 matrix approach in [5] to the Fourier transform domain (FTD). Significant advantage is gained by working in the FTD because Green's function convolution integrals for determination of field quantities due to current sources are converted into algebraic products. The FTD process in addition treats asymmetrical conductor lines in the same way as it treats symmetrical conductor lines. From the Green's function G the procedure for determining the propagation constant γ is provided for the method of moments numerical technique assisted by identical expansion and test basis functions (Galerkin approach).

Normal Mode Field Solution

Each layer has four eigenfunction field solution sets. Superposition of these four normal mode sets of field components constitutes the actual total field solution obeying all boundary conditions (BC). Time harmonic, plane guided wave solutions proportional to exp $(j\omega t - \gamma z)$ are assumed. Insertion of the time harmonic nature of the plane wave into Maxwell's two curl equations creates the single sourceless matrix equation $\tilde{L}_T \tilde{V}_L = j\omega \tilde{V}_R$, in the FTD with tildes denoting FTD variables. Here

$$\tilde{L}_T = \left[\begin{array}{c|c} 0 & \tilde{L}_1 \\ \hline -\tilde{L}_1 & 0 \end{array}\right], \qquad \tilde{L}_1 = \begin{bmatrix} 0 & \gamma & d/dy \\ -\gamma & 0 & -jk_x \\ -d/dy & jk_x & 0 \end{bmatrix}. \qquad \text{(1a,b)}$$

Fourier transformed electromagnetic fields $\tilde{V}_L$ and $\tilde{V}_R$ are $\tilde{V}_L = [\tilde{E}_x \ \tilde{E}_y \ \tilde{E}_z \ \tilde{H}_x \ \tilde{H}_y \ \tilde{H}_z]^T$, $\tilde{V}_R = [\tilde{D}_x \ \tilde{D}_y \ \tilde{D}_z \ \tilde{B}_x \ \tilde{B}_y \ \tilde{B}_z]^T$ (superscript T = transpose). One dimensional Fourier transform pair $(f,\tilde{f})$ is defined as

$$\tilde{f}(k_x,y) = \int_{-\infty}^{\infty} f(x,y) e^{-jk_x x} dx, \quad f(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{f}(k_x,y) e^{jk_x x} dk_x. \qquad \text{(2a,b)}$$

where $f(x,y)$ is any real space variable. The medium of each layer can be characterized by a single 6 × 6 constitutive tensor $\hat{M}$ in the FTD $\tilde{V}_R = \hat{M}\tilde{V}_L$, where

$$\hat{M} = \left[\begin{array}{c|c} \hat{\epsilon} & \hat{\rho} \\ \hline \hat{\rho}' & \hat{\mu} \end{array}\right]. \qquad (3)$$

$\hat{\epsilon}$ and $\hat{\mu}$ are respectively the permittivity and permeability tensors. $\hat{\rho}$ and $\hat{\rho}'$ tensors are responsible for optical activity. The equation which must be solved for the normal mode field vectors is $\tilde{L}_T \tilde{V}_L = j\omega \hat{M} \tilde{V}_L$. It is then converted into the following form:

$$\frac{1}{j\omega}\frac{d\phi}{dy} = R\phi. \qquad (4)$$

ϕ is the four element column vector in the FTD having only interface tangential field components, $\phi = [\tilde{V}_1 \ \tilde{V}_3 \ \tilde{V}_4 \ \tilde{V}_6]^T = [\tilde{E}_x \ \tilde{E}_z \ \tilde{H}_x \ \tilde{H}_z]^T$.

Translate y into the mth local layer shifted coordinate system $y'_m = y - \Sigma_{j=1}^{m-1} h_j$, where h_j is the jth layer thickness and $y = 0$ corresponds to an interface. Solutions to (4) in the y'_m coordinate system can be written as $\phi(y'_m) = \exp(jk_y y'_m)\phi(0)$. Putting this into (4) produces

$$\left\{\frac{k_{yi}}{\omega} I - R\right\}\phi_i(0) = 0, \qquad \det\left\{\frac{k_{yi}}{\omega} I - R\right\} = 0 \qquad \text{(5a,b)}$$

Equation (5b) generates four k_y eigenvalues k_{yi}. These k_{yi} values are placed in (5a) to find the individual $\phi_i(0)$ normal mode vectors at $y'_m = 0$. The normal mode vector $\phi_i^m(y'_m)$ at $y'_m > 0$ is found from $\phi_i^m(0)$ by multiplication with a 4 × 4 matrix characterizing the mth layer medium:

$$\phi_i^m(y'_m) = P^m(y'_m)\phi_i^m(0). \qquad (6)$$

Here $P^m(y'_m) = \Psi^m(0)K^m(y'_m)\Psi^m(0)^{-1}$, $K_{ij}^m = \delta_{ij}\exp(jk_{yi}^m y'_m)$, and $\Psi^m(0) = [\phi_1^m(0) \ \ \phi_2^m(0) \ \ \phi_3^m(0) \ \ \phi_4^m(0)]$. The superscript m in (6) emphasizes the specialization of the eigenvector solution to the mth layer. $\Psi^m(0)$ is a 4 × 4 matrix constructed out of the four normal mode vectors.

Half-Open Guided Wave Structure

Figure 1 shows the structure of the waveguiding layered configuration. It has top and bottom perfectly conducting ground planes. Layers are characterized by thicknesses h_j, there being a total of $n + 1$ layers. There are n interfaces between the layers, and a total of n interface

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surface currents $\vec{J}_s^j$. These surface currents are line sources and may be thought of in an abstract sense as representing perfectly conducting lines positioned at the interfaces. Each layer is characterized by a single constitutive tensor $\hat{M}_j$ [see (3)] for the jth layer, there being a total of $n + 1$ tensors for all layers. The open structure to be analyzed is mathematically and physically created by letting $h_{n+1} \to \infty$. In the $(n + 1)$th layer only $+y$ outward propagating (or decaying) waves are desired in this limit, and this fact is utilized later to find the Green's function.

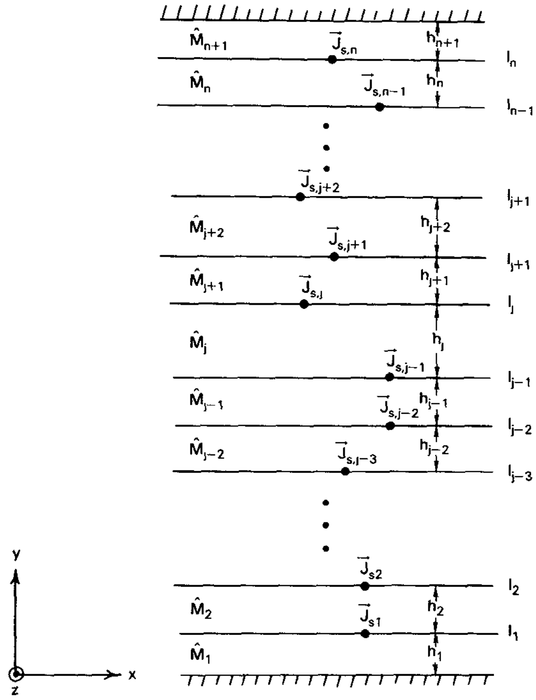

Fig. 1 — Cross-section of guided wave structure containing $n + 1$ layers of thickness h_j, each characterized by a 6×6 constitutive tensor $\hat{M}_j$. There are n interface impulse surface current line sources $\vec{J}_{s,j}$. The whole structure is bounded by two perfectly conducting electric walls.

The normal mode or eigenvector solutions found must be superpositioned to represent the actual (total) field solution in each layer, $\Lambda^m(y'_m) = \Sigma_{i=1}^4 e_i \phi_i^m(\phi'_m)$. The mapping of $\Lambda^m(0)$ into $\Lambda^m(y'_m)$ which is essential to the derivation below is found using (6):

$$\Lambda^m(y'_m) = \sum_{i=1}^{4} e_i P^m(y'_m)\phi_i^m(0) = P^m(y'_m) \sum_{i=1}^{4} e_i\phi_i^m(0) = P^m(y'_m)\Lambda^m(0). \qquad (7)$$

Four element column vector Λ^m is convenient to use for applying interface or boundary conditions since the BC's can be expressed solely in terms of tangential field components. Electric field components Λ_1^m and Λ_2^m ($\tilde{E}_x^m$ and $\tilde{E}_z^m$) at the mth interface are continuous. Magnetic field components Λ_3^m and Λ_4^m ($\tilde{H}_x^m$ and $\tilde{H}_z^m$) are discontinuous. The BC's at the interfaces in matrix notation are $\Lambda^m(h_m^+) = \Lambda^m(h_m^-) + [0 \;\; 0 \;\; -\tilde{J}_{sz}^m \;\; \tilde{J}_{sx}^m]^T$, where the superscript T on the last vector means transpose.

$\Lambda^m(y'_m)$ in the mth layer is found by starting at $y'_1 = 0$ and proceeding through each layer using (7), accounting for field match or mismatch along the way at successive interfaces by enlisting BC's. The result of this procedure is

$$\Lambda^m(y'_m) = F^{m,m}\Lambda^1(0) + \sum_{k=1}^{m-1} F^{m,m-k}\,[0 \;\; 0 \;\; -\tilde{J}_{sz}^k \;\; \tilde{J}_{sx}^k]^T, \qquad (8)$$

$$F^{m,l}(y'_m) = \sum_{j=m-l+1}^{m} P^j(h'_j). \qquad (9)$$

In (9), $h'_j = h_j$, the jth layer thickness, except for the mth layer where $h'_j = y'_m$. $F^{m,l}$ in (9) has a simple but elegant physical interpretation. It is a mapping prefactor operator which takes the quantity to its right and pulls it through l layers until it arrives at y'_m in the mth layer. Applied to (8), the operator with $l = m$ takes the field vector $\Lambda^1(0)$ and draws it through all m layers to the position y'_m in the final mth layer. When $l = m - k$ as in the second operator acting on the kth discontinuity BC, the operator pulls the BC sitting on the kth interface, which is on top of the kth layer and on the bottom of the $k + 1$ layer, through the remaining $m - k$ layers.

$\Lambda^1(0)$ is unknown in (8) and needs to be determined so that $\Lambda^m(y'_m)$ is uniquely specified. The first two components of $\Lambda^1(0)$ are $\Lambda_1^1(0) = \Lambda_2^1(0) = 0$ due to the ground plane BC. $\Lambda_3^1(0)$ and $\Lambda_4^1(0)$ are determined by using (8) to connect the ground plane at $y = y'_1 = 0$ and the $(n + 1)$th side of the nth interface. Invoking (8),

$$\Lambda^{n+1}(0) = F^{n+1,n+1}(0)\Lambda^1(0) + \sum_{k=1}^{n} F^{n+1,n+1-k}(0)\,[0 \;\; 0 \;\; -\tilde{J}_{sz}^k \;\; \tilde{J}_{sx}^k]^T. \qquad (10)$$

Here one notes that $F^{n+1,n+1-k}(0) = F^{n,n-k}(h_n)$ for $n \geqslant k \geqslant 0$. $\Lambda^{n+1}(0)$ must be specified in order to solve (10) for the necessary $\Lambda^1(0)$ components. First the normal modes comprising $\Lambda^{n+1}(0)$ need to be obtained. This layer is isotropic with [see (3)] $\hat{\epsilon} = \epsilon_{n+1}I$, $\hat{\mu} = \mu_{n+1}I$, $\hat{\rho} = \hat{\rho}' = 0$. Note that $\tilde{V}_i = \Sigma_{j=1}^6 a'_{ij}\,(1 - \delta_{2,j})\,(1 - \delta_{5,j})\,\tilde{V}_j/D_a$, $i = 2, 5$, with $D_a = \epsilon_{n+1}\mu_{n+1}$. Defining a'_2 and a'_5 as the vectors containing their respective a'_{ij}, $i = 2$ or 5, $j = 1, 2, 4$ and 6, $a'_2 = [0 \;\; 0 \;\; j\gamma\mu_{n+1}/\omega \;\; -k_x\mu_{n+1}/\omega]^T$ and $a'_5 = [-j\gamma\epsilon_{n+1}/\omega \;\; k_x\epsilon_{n+1}/\omega \;\; 0 \;\; 0]^T$.

We find R to be

$$R = \begin{bmatrix} 0 & 0 & -\dfrac{j\gamma k_x}{\omega^2\epsilon_{n+1}} & -\left(\mu_{n+1} - \dfrac{k_x^2}{\omega^2\epsilon_{n+1}}\right) \\ 0 & 0 & \left(\mu_{n+1} + \dfrac{\gamma^2}{\omega^2\epsilon_{n+1}}\right) & \dfrac{j\gamma k_x}{\omega^2\epsilon_{n+1}} \\ \dfrac{j\gamma k_x}{\omega^2\mu_{n+1}} & \left(\epsilon_{n+1} - \dfrac{k_x^2}{\omega^2\mu_{n+1}}\right) & 0 & 0 \\ -\left(\epsilon_{n+1} + \dfrac{\gamma^2}{\omega^2\mu_{n+1}}\right) & -\dfrac{j\gamma k_x}{\omega^2\mu_{n+1}} & 0 & 0 \end{bmatrix} \qquad (11)$$

Normal mode eigenvalues k_{yi} are found by placing (11) into (5b). The dispersion equation $\{k_{yi}^2 - [k_{n+1}^2 - k_x^2 + \gamma^2]\}^2 = 0$ results with $k_{n+1}^2 = \omega^2\epsilon_{n+1}\mu_{n+1}$. This equation produces two distinct eigenvalues, or a total of four normal modes, two being degenerate. Placing k_{yi} into (5a) and using three of the four implied equations produces the eigenvectors $\phi_i^{n+1}(0)$. We choose k_{yi}, $i = 1$ or 2, to be the distinct eigenvalues, and denote the two different eigenvectors associated with each i by subscripts a and b. Because the $(n + 1)$th layer is semi-infinite, only the outward propagating (or decaying) wave in the $+y$ direction is displayed below.

$$\phi_{1a}^{n+1}(0) = \left[\frac{\omega\mu_{n+1}k_{y1}^{n+1}}{k_{n+1}^2 + \gamma^2} \;\;\; 0 \;\;\; -\frac{j\gamma k_x}{k_{n+1}^2 + \gamma^2} \;\;\; 1\right]^T;$$

$$\phi_{1b}^{n+1}(0) = \left[-\frac{j\gamma k_x}{k_{n+1}^2 + \gamma^2} \;\;\; 1 \;\;\; -\frac{\omega\epsilon_{n+1}k_{y1}^{n+1}}{k_{n+1}^2 + \gamma^2} \;\;\; 0\right]^T. \qquad (12)$$

Notice that this degenerate normal mode solution consists of TM_z and TE_z forms. Since the medium is isotropic, the solution could have been resolved into other TM_n and TE_n forms where n = rectangular axis.

Equations (12) are used to construct the total field vector at $y'_{n+1} = 0$, $\Lambda^{n+1}(0) = A_{n+1}\phi_{1a}^{n+1}(0) + B_{n+1}\phi_{1b}^{n+1}(0)$. Equating this to (10) creates a single vector equation comprised of four linear algebraic equations in four unknowns A_{n+1}, B_{n+1}, $\Lambda_3^1(0)$ and $\Lambda_4^1(0)$. $\Lambda_3^1(0)$ and $\Lambda_4^1(0)$ are solved as

$$\Lambda_j^1(0) = \sum_{i=1}^{4} C_i b_{ji};\; j = 3, 4 \;(or\; x,z), \qquad (13a)$$

$$C_i = \sum_{m=1}^{n-1} (-F_{i3}^{n,n-m}\tilde{J}_{sz}^m + F_{i4}^{n,n-m}\tilde{J}_{sx}^m) - \delta_{i3}\tilde{J}_{sz}^n + \delta_{i4}\tilde{J}_{sx}^n, \qquad (13b)$$

$$b_{ji} = -(-1)^j \frac{1}{D_o} \det\,[\bar{\phi}_{\bar{i}a} \;\; \bar{\phi}_{\bar{i}b} \;\; -\bar{F}_{\bar{i},7-j}], \qquad (13c)$$

$$D_o = \det\,[\phi_{1a} \;\; \phi_{1b} \;\; -\bar{F}_3 \;\; -\bar{F}_4], \qquad (13d)$$

$$\bar{\phi}_{\bar{i}(a,b)} = \left[\phi_{1j(a,b)}^{n+1} \;\; \phi_{1k(a,b)}^{n+1} \;\; \phi_{1l(a,b)}^{n+1}\right](0)^T; \qquad (14a)$$

$$\bar{F}_{\bar{i},(3,4)} = \left[F_{j(3,4)}^{n,n} \;\; F_{k(3,4)}^{n,n} \;\; F_{l(3,4)}^{n,n}\right]^T, \qquad (14b)$$

$$\bar{F}_{(3,4)} = \left[F_{1(3,4)}^{n,n} \;\; F_{2(3,4)}^{n,n} \;\; F_{3(3,4)}^{n,n} \;\; F_{4(3,4)}^{n,n}\right]^T. \qquad (14c)$$

In (14) the component subscripts j,k,l are cyclic, exclude i (notation $\bar{i}$), and equal 1 through 4. Equation (13a) is inserted into (8) so that $\Lambda^m(y'_m)$ is determined. The $\tilde{E}_x^m(h_m)$ and $\tilde{E}_z(h_m)$ electric field components are then extracted out of the resulting (8) and related to all the interface surface current vectors (or surface current components). The identical procedure is carried out for each interface set of electric field components, yielding a total of n interface electric field component sets related to n interface surface current vectors.

Dyadic G relates these two sets, $U = GW$, given by

$$U = \begin{bmatrix} \tilde{E}_1^1 \\ \tilde{E}_2^1 \\ \cdot \\ \cdot \\ \cdot \\ \tilde{E}_1^n \\ \tilde{E}_2^n \end{bmatrix} = \begin{bmatrix} S^{11} & S^{12} & \cdots & S^{1n} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ S^{n1} & S^{n2} & \cdots & S^{nn} \end{bmatrix} \begin{bmatrix} \tilde{J}_{s1}^1 \\ \tilde{J}_{s2}^1 \\ \cdot \\ \cdot \\ \cdot \\ \tilde{J}_{s1}^n \\ \tilde{J}_{s2}^n \end{bmatrix} = GW. \quad (15)$$

U and W are electric field component and interface current component vectors of size $2n$. Subscript indices on their field or current elements above denote $x(j = 1)$ or $z(j = 2)$ components. Elements of U are u_l, and of W, w_l. G is a $2n \times 2n$ matrix with each 2×2 S^{mp} submatrix element given by $(i,\ j = 1, 2)$

$$S_{ij}^{mp} = T_{ij}^{mp} + F_{i,5-j}^{m,m-p}, \quad (16)$$

$$T_{ij}^{mp} = \begin{cases} -(-1)^j \sum_{k=1}^{4} F_{k,5-j}^{n,n-p} (b_{xk} F_{l3}^{m,m} + b_{zk} F_{l4}^{m,m}), & p = 1, 2, \ldots, n-1 \quad (17a) \\ -(-1)^j (b_{x,5-j} F_{l3}^{m,m} + b_{z,5-j} F_{l4}^{m,m}), & p = n. \quad (17b) \end{cases}$$

In (16) the second term is dropped if $p \geqslant m$.

Propagation constant γ is obtaining from (15), the starting point of a Galerkin process, by using basis (expansion) and test functions. Represent each element of the W vector as $w_l(k_x, \gamma) = \Sigma_{s=1}^{N_l} q_{ls} w_{ls}(k_x, \gamma)$ where w_{ls} are the FTD basis functions which may be chosen to be a complete set and q_{ls} are weight coefficients. Multiply rows 1 through $2n$ of w_l by respectively $w_{ls'}^*$ through $w_{(2n)s'}^*$ with $s' = 1, 2, \ldots, N_l$ creating N_l equations for the lth row. The total number of equations will be $N = \Sigma_{l=1}^{2n} N_l$. Integrate these equations over reciprocal k_x space (drop the $1/2\pi$ factor) obtaining

$$Y = S_x Q, \quad (18)$$

where

$$S_x = \begin{bmatrix} X^{11} & X^{12} & \cdots & X^{1(2n)} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ X^{(2n)1} & X^{(2n)2} & \cdots & X^{(2n)(2n)} \end{bmatrix}, \quad (19)$$

with $Q = [q_{11} \quad q_{12} \quad \ldots \quad q_{1N_1} \quad q_{21} \quad \ldots \quad q_{(2n)N_{2n}}]^T$, and

$$X_{s's}^{ij} = \int_{-\infty}^{\infty} w_{is'}^* G_{ij} w_{js} dk_x, \qquad Y_{is'} = \int_{-\infty}^{\infty} u_i w_{is'}^* dk_x. \quad (20a,b)$$

Each $X_{s's}^{ij}$ is the $s's$th element of the X^{ij} submatrix S_x. Using FTD properties,

$$Y_{is'}(\gamma) = \int_{-\infty}^{\infty} u_i(k_x, \gamma) w_{is'}^*(k_x, \gamma) dk_x = 2\pi \int_{-\infty}^{\infty} u_i(x, \gamma) w_{is'}^*(x, \gamma) dx. \quad (21)$$

$u_i(x, \gamma)$ and $w_{is'}(x, \gamma)$ are the electric field and current distributions on the interfaces. If one assumes perfect conductors on the interfaces, $w_{is'} = 0$ when $u_i \neq 0$ and the converse. This complementary nature of interface fields and currents makes all $Y_{is'} = 0$. It is not necessary for field or current symmetry with respect to the x-axis to hold in order to assure that the left hand side of (21) is zero. The homogeneous set of equations in (18) for the q_{ls} coefficients requires det $(S_x) = 0$ for a solution to exist, generating the characteristic dispersion equation in γ to be solved. It produces an infinite set of γ_l eigenvalues, each a complex number $\gamma_l = \alpha_l + j\beta_l$. α_l is the attenuation constant and β_l the phase propagation constant. Phase velocity of the lth eigenvalue is given by $v_{pl} = \omega/\beta_l$ where ω is the radian frequency. Electromagnetic tangential fields can be obtained from (8) in any layer by inserting the calculated γ_l. The transverse field components $\tilde{E}_y^m$ and $\tilde{H}_y^m$ are $[\tilde{E}_y^m \quad \tilde{H}_y^m]^T = [a_2^T \quad a_5^T]^T \Lambda^m$ where a_i^T are transposes of the vectors discussed earlier.

For a closed structure h_{n+1} is finite, and analysis similar to the open structure is performed. A equation like (10) results but with $y'_{n+1} = 0$ replaced by h_{n+1} in the $(n + 1)$th layer. Applying BC's at $y'_{n+1} = h_{n+1}$ imposes $\Lambda_1^{n+1}(h_{n+1}) = \Lambda_2^{n+1}(h_{n+1}) = 0$. Putting these BC's into the modified (10) yields an equation for the $\Lambda^1(0)$ components:

$$[0 \quad 0 \quad \tilde{H}_x(h_{n+1}) \quad \tilde{H}_z(h_{n+1})]^T = F^{n+1,n+1} [0 \quad 0 \quad \tilde{H}_x^1(0) \quad \tilde{H}_z^1(0)]^T + \sum_{k=1}^{n} F^{n+1,n+1-k} [0 \quad 0 \quad -\tilde{J}_{sz}^k \quad \tilde{J}_{sx}^k]^T \quad (22)$$

Using only the first two rows of (22),

$$\Lambda_{i+2}^1(0) = \tilde{H}_i^1(0) = \sum_{j=1}^{2} \sum_{k=1}^{n} I_{ij}^k \tilde{J}_{sj}^k; \quad i,j = 1, 2(x,z), \quad (23a)$$

$$I_{ij}^k = (-1)^{i+j} [F_{2,5-i}^{n+1,n+1} F_{1,5-j}^{n+1,n+1-k} - F_{1,5-i}^{n+1,n+1} F_{2,5-j}^{n+1,n+1-k}]/D_c, \quad (23b)$$

where D_c is the determinant of a 2×2 matrix with elements $F_{i,j+2}^{n+1,n+1}$, $i,\ j = 1,\ 2$. Using (23), one finds that

$$S_{ij}^{mp} = T_{ij}^{mp} - (-1)^j F_{1,5-j}^{m,m-k}, \quad (24)$$

$$T_{ij}^{mk} = F_{l3}^{m,m} I_{ij}^k + F_{l4}^{m,m} I_{2j}^k, \quad (25)$$

Equations (24) and (25) are used to create the dyadic G as is (15).

Conclusion

The great value of the matrix method covered in this paper is that it permits a systematic approach for solving most planar guided wave problems. Methodology is so general as to afford solutions to the most complex anisotropic layered problems, as well as simpler problems. The starting point of the method is the specification of constitutive macroscopic tensor $\hat{M}_i$ characterizing each layer i. For example, in a relatively thick semiconductor layer where the bulk $\hat{M}_i$ can be used, $\hat{M}_i$ would arise from various microscopic transport effects such as impurity coulomb scattering, alloy scattering, carrier-carrier scattering via coulomb and exchange interactions, and intravalley and intervalley scattering. Converting the above semiconductor transport effects from the microscopic to the macroscopic level is well understood and regularly done. For layers which are narrow and leads to separation of carrier energy levels in the transverse or y-direction, this two dimensional effect may play a noticable role in altering the scattering behavior through change of carrier quantum mechanical wavefunctions Ψ_c. If the layer walls are considered impenetrable to Ψ_c, then size effect, energy level splitting on the order of single particle $\epsilon_n = (n\hbar\pi/h_n)^2/2m^*$ can be expected where $\hbar$ is Planck's constant and m^* the carrier effective mass. The layer size effect discussed here is also related to similar two dimensional quantum mechanical phenomena, namely quantum mechanical well creation and transport effects in metal-insulator-semiconductor (MIS) structures and devices [6].

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